

## THE FDTD DIAKOPTICS METHOD

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### ABSTRACT

A finite-difference time-domain (FDTD) Diakoptics method is developed in this paper. Sequential and parallel algorithms are provided to connect cascaded segments and to realize the modular computation of large circuits. The proposed methods are applied to a one-dimensional case to confirm their validity by comparison with the results from Microwave SPICE (MWSPICE).

### INTRODUCTION

For the rigorous analysis of wide-band and nonlinear microwave integrated circuits, the lumped circuit simulation is directly included into the time-domain field analysis methods, such as the transmission line matrix (TLM) [1], the spacial network (SNW) [2], and the finite-difference time-domain (FDTD) [3]. These methods can simulate the entire circuit in one step and include all the mutual interactions among circuit elements.

To maintain numerical accuracy at high frequencies, these time-domain methods require a fine spatial discretization that results in a large computer storage. For a large and complicated circuit, this memory requirement for circuit simulation is often not practical. One possible solution is the time-domain Diakoptics method. The time-domain Diakoptics idea was first introduced in TLM [4]-[6]. The present paper will propose the time-domain Diakoptics method in FDTD. The implementation of this idea in FDTD is illustrated first. Sequential ("impedance transformation" type) and parallel algorithms (cascaded "matrix" type) for the Diakoptics of cascaded segments are then demonstrated.

### THE TIME-DOMAIN DIAKOPTICS

The time-domain Diakoptics idea originates from the linear system theory. The system output  $Y(n)$  of a one-port linear passive system can be determined from the convolution operation between the impulse response  $h(n)$  and the system input  $X(n)$ . This means that the whole one-port linear passive system can be replaced by its impulse response  $h(n)$ . Similarly, a multi-port linear passive region in the field calculation can be replaced by an impulse response matrix  $[g]$ , Fig. 1, that is similar to a time-domain Green's function [7]. The multi-port convolution is defined in Eq.(1).

$$Y_m(k) = \sum_{n=1}^N \sum_{k'=0}^k g(m,n,k-k') X_n(k') \quad (1)$$

Where

$g(m,n,k')$  is the impulse response at port  $m$  at  $t=k'$  due to the unit excitation at port  $n$  at  $t=0$ .

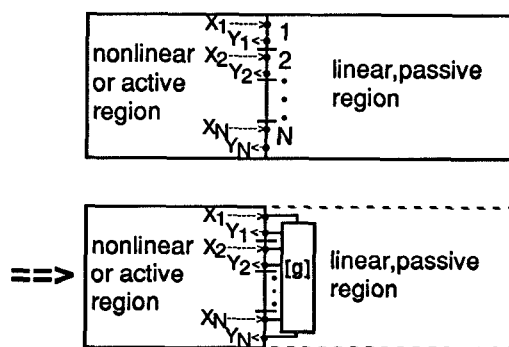


Figure 1. A multi-port linear passive region is replaced by its impulse response matrix  $[g]$ .

## THE FDTD DIAKOPTICS METHOD

The principle of the FDTD Diakoptics method is demonstrated by a one-port connected circuit in Fig. 2, although the method can be extended to two or three dimensional structures. An infinitely long parallel-plate transmission line is loaded with lumped elements, and only TEM wave exists along this line. The use of one-dimensional model makes it possible that the results can be compared with those by a conventional time-domain circuit simulator, like MWSPICE.

This structure is divided into two sections in Fig. 3. The left-hand section is a nonlinear active region and the other section is a linear passive region. Unlike TLM in which the waves of different directions are separately defined, the field variables in FDTD represent the total field. Hence two overlapped cells are used on the segmentation boundary to represent the input and output variables in the convolution operation [8].  $X_1^t$  and  $Y_1^t$  is the total tangential e-field near the segmentation boundary. After an impulse is injected into the right-hand section (RHS) at the position of  $X_1^t$ , the impulse response  $h_1^t$  can be found at the position of  $Y_1^t$ . During the time iterations of FDTD Diakoptics method, the field of a boundary cell is calculated from the convolution operation with  $h_1^t$  which represents RHS. With the aid of Eq.(1), this algorithm can be applied to a multi-port segmentation boundary.

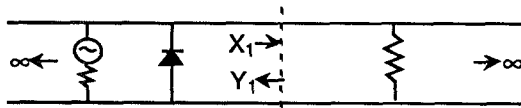


Figure 2. An infinitely long parallel-plate transmission line loaded with lumped elements.

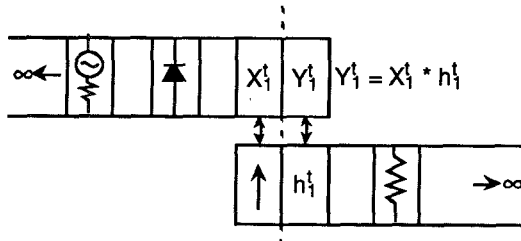


Figure 3. The implementation of FDTD Diakoptics on a one-port connected circuit.

## THE SEQUENTIAL FDTD DIAKOPTICS ALGORITHM

From the previous section, a circuit can be divided into a linear passive part and a nonlinear active part, and connected through an impulse response matrix. For a very large linear passive region, a further segmentation is helpful. To this end, a sequential algorithm is proposed to connect cascaded segments.

A multi-layered circuit in Fig.4 is used to demonstrate the sequential algorithm. Sections (3) and (4) in Fig.4 are cascaded sections. Two outer sections, (1) and (4), are calculated first. During step 2, when the inner section (3) is simulated by FDTD, a convolution operation is performed on the boundary between (3) and (4). Following this procedure, cascaded sections (3) and (4) is represented by  $h_d^t$ . If there are more cascaded sections, this process is repeated. Finally, step 3 is solved.

Using the sequential FDTD Diakoptics algorithm, the transient response of the circuit in Fig.4 is plotted in Fig.5 and compared with simulated results from Microwave SPICE. The agreement is so good that they cannot be distinguished on the figure. If section (3) is modified, then only steps 2 and 3 need to be recalculated. The computation effort on sections (1) and (4) in step 1 need not be repeated. Fig. 6 shows the agreement of the circuit response calculated by the cascaded segmentation and MWSPICE.

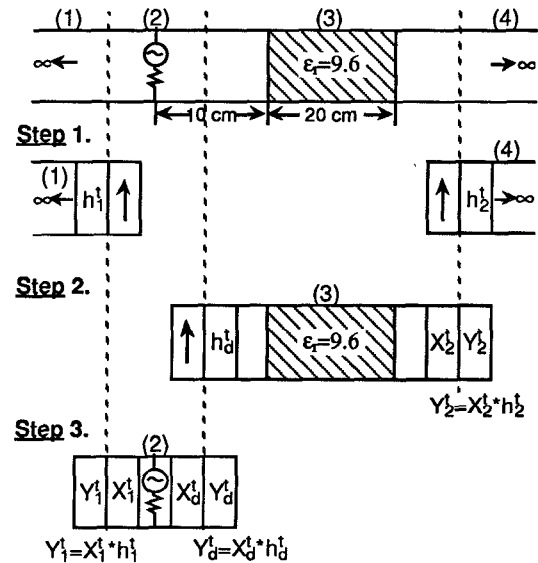


Figure 4. A multi-layered circuit is analyzed by the sequential FDTD Diakoptics algorithm.

## THE PARALLEL FDTD DIAKOPTICS ALGORITHM

Further improvement in computation is obtained by a parallel algorithm. The parallel algorithm [6] can simulate each section by FDTD separately. When one section is modified, no other section needs to be recalculated. This algorithm is illustrated in Fig 7. Section (3) is treated as a two-port segment that needs to be simulated twice, one for the wave propagating to the right denoted as "+", and one for the wave to the left denoted by "-". The reflected wave  $J_{11}$  from the resistor is needed to calculate  $G$  in step 2. To this end, section (4) is replaced by a semi-infinite structure on the bottom of step 1. This process gives rise to  $J_{11}^+$ .  $J_{11}^-$

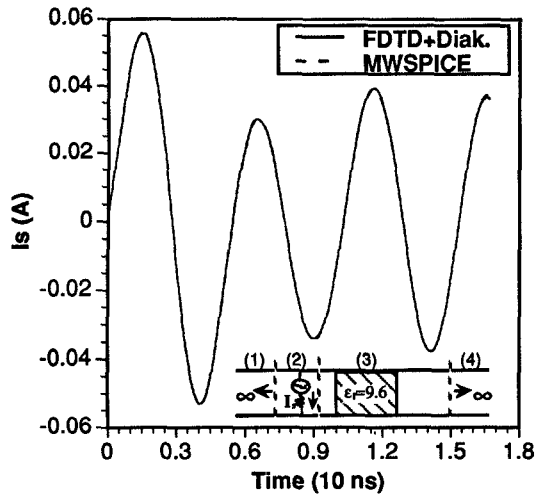


Figure 5. The transient response of the circuit in Fig. 4 ( $V_s = -0.1 \sin(2\pi ft)$ ,  $f = 200$  MHz,  $R_s = 0.5 \Omega$ , Cell size =  $1 \text{ cm}^2$ )

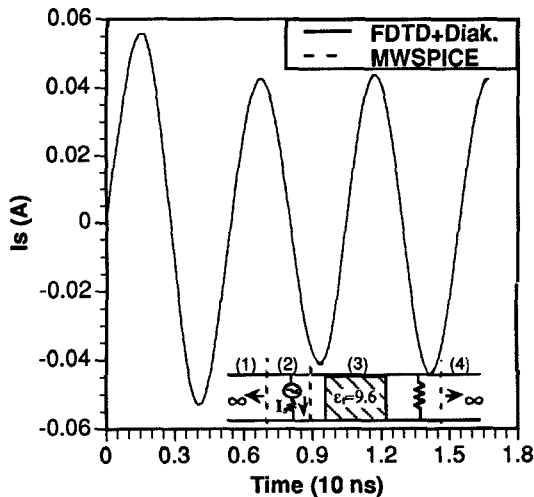


Figure 6. The transient response of a modified circuit from Fig. 4 ( $V_s = -0.1 \sin(2\pi ft)$ ,  $f = 200$  MHz,  $R_s = 0.5 \Omega$ ,  $R_L = 2.5 \Omega$ , Cell size =  $1 \text{ cm}^2$ )

can be obtained from  $J_{11}^+$  and  $J_{11}^-$ . Since there is no multiple reflection between sections (3) and (4), only two terms are required in the calculation of total impulse response  $G$  for sections (3) and (4). Fig. 8 shows the agreement between the circuit response simulated by the parallel algorithm and the MWSPICE.

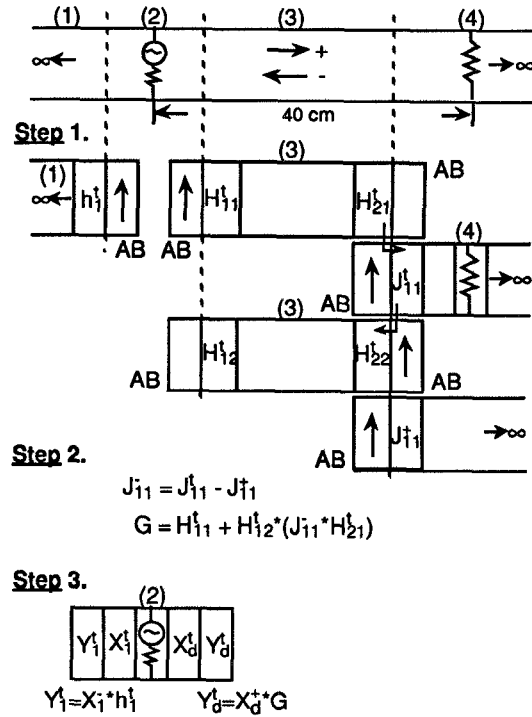


Figure 7. Application of the parallel FDTD Diakoptics algorithm to a circuit.

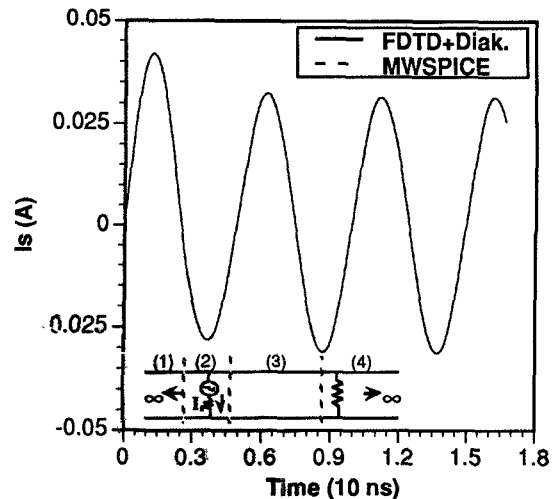


Figure 8. The transient response of the circuit in Fig 7. ( $V_s = -0.1 \sin(2\pi ft)$ ,  $f = 200$  MHz,  $R_s = 0.5 \Omega$ ,  $R_L = 2.5 \Omega$ , Cell size =  $1 \text{ cm}^2$ )

## CONCLUSION

The FDTD Diakoptics method is developed. Sequential and parallel algorithms are implemented in FDTD to make the computation more efficient for a large circuit. When such a large circuit is modified, only a few segments need to be recalculated, while the mutual interaction is preserved. This means that the repetitive computation of large structures is avoided and the modular computation of large circuits is realized.

## ACKNOWLEDGMENTS

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